Geometric Morphometrics and Geological Form-Classification Systems

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Abstract.—Many areas of geological inquiry involve the description and/or comparison of shapes. While various morphometric tools have long been available to facilitate these types of comparisons, by far the most common approach to such form-classification has been via the creation of a semi-quantitative scale of morphological exemplars, type specimens, etc. to which unknown structures, objects, or specimens can be referred. Such form-scales are ubiquitous—either in terms of text-based descriptions or illustration sets—throughout the geological literature. However, students, and even experienced geologists, often have difficulty using such scales or achieving consistent results. Investigations of three such scales drawn from the fields of sedimentology, paleontology, and geomorphology using the analytical tools of geometric morphometrics suggests that one reason for this difficulty is that exemplars drawn from sets of real objects often exhibit shape differences other than those under consideration by the scale, or whose class boundaries are insufficiently documented/described. Herein strategies are developed that employ the shape-space ordination and modeling capabilities of eigenshape analysis to correct these deficiencies and devise sets of new, more representative, and easier to use shape classification systems. By employing these approaches, augmented where necessary with formal statistical analyses, geologists can improve the sophistication, accuracy, and reproducibility of their morphological inferences and, in so doing, improve the reliability of their hypotheses tests.

Keywords: Geometric morphometrics, geology, sedimentology, paleontology, geomorphology

Introduction

Geological analysis involves, in many instances, a comparison between forms. Obvious examples are form-based comparisons between fossil species made by paleontologists in solving a stratigraphical problems or between landforms by geomorphologists in interpreting the history of a landscape. Less obvious, but just as important, examples might include the comparison of fold-shapes by structural geologists, the comparison of mineral grain shapes by mineralogists, the comparison of fluvial bedform profiles by sedimentologists, or even the comparison of seismogram traces by geophysicists. In all of these cases (and many more) the outcome of the analysis-interpretation hinges of the manner, acuity, and objectivity with which such geometric comparisons can be made.

While the field of geometry—especially analytic geometry—is obviously associated with shape studies, most geological applications of quantitative shape analysis trace their origins to the disciplines of pattern recognition or to morphometrics. The distinction between these three concepts of shape-based investigations is important in that they concern themselves with different, but related, types or domains of analyses. In its canonical form, geometry is the study of shape and the properties of shape. Geometry is what connects pattern recognition and morphometrics to the natural world.

Pattern recognition begins with a geometric representation of an object or a scene (usually in the form of a digital image) and attempts to find and identify a pattern within the image frame (Gonzalez and Wintz 1977; Baxes 1984). A typical pattern recognition problem involves elements of image processing, image segmentation, and the geometry-based comparison of image segments with sets of target images or primitives. The best modern pattern recognition systems also incorporate an ability to construct new target images out of pervious experience and so incorporate an element of machine 'learning'.

Morphometrics, though often mistaken for geometry and pattern recognition, is actually the study of covariances with shape (Bookstein 1991). Perhaps slightly more practically, it can be defined as the study of covariances between shape representations and other associated or causal variables. In other words, whereas morphometrics is not involved with the identification of objects per se (= pattern recognition), it is concerned with the degree to which other variables (e.g. time, space, composition, ecology, phylogeny) are related to shape variation within a sample or population and the nature of that relationship. As such, morphometrical data analysis strategies can be used to address a far wider range of shape-related problems than either geometry or pattern recognition.

The morphometric approach to hypothesis testing is based implicitly on the idea of ordination. Ordination is the process of arranging objects or classes in an *n*-dimensional space such that inter-object distances reflect assessments of similarity or dissimilarity (Sneath and Sokal 1973; Chatfield and Collins 1980; Howard 1991). Unlike geometry or pattern recognition, the inter-object ordinations of morphometric analysis provides this approach with its ability to evaluate hypotheses of association, correlation, and (ultimately) causation. However, it must be borne in mind that—for all but the simplest morphologies—the hypothesis under examination is not whether extraneous variable(s) cause or covary with the observed objects' morphological variation. Rather, it is whether the extraneous variable(s) cause or covary with the aspects of the objects' morphological variation (= the aspect(s) of its geometry) actually measured or extractable from the data matrix submitted to analysis, see MacLeod 1999). Confusion over this issue inevitably leads to erroneous interpretations of morphometric results.

Prior to the 1980's the most popular forms of morphometrical analysis (= multivariate morphometrics) relied on matrices of linear distances to ordinate objects within an abstract shape space or size-shape space. While, these scalar data were sufficient to identify intra-sample trends in morphological variation, the nature of such data—specifically the lack of an ability to accurately model the shape transformations represented graphical ordinations of latent vector equations—was problematic. This deficiency made it difficult to confirm the interpretations of ordinations and communicate results to non-quantitative audiences.

Morphometric data analysis methods that possessed this representational-modeling capability did exist (e.g., biorthogonal analysis, Bookstein 1978; radial fourier analysis, Christopher and Waters 1974; θ–ρ analysis, Benson 1967; Siegel and Benson 1982), but these did not prove popular because of their complexity or the lack of easy-to-use software. Intermediate, hybrid methods that mixed various graphic representation strategies with eigenvector-based multivariate ordinations began to appear in the early 1980's (e.g., truss analysis, Strauss and Bookstein 1982; eigenshape analysis, Lohmann 1983; distance-interval analysis, Klapper and Foster 1986). Then, in 1984–1991 a conceptual synthesis occurred that united several (up to then) different methodological research programs into a new 'geometric morphometrics' (Kendall 1984; Bookstein 1986, 1991, 1993; Rohlf and Bookstein 1990; Goodall 1991).

While there are many strands to the morphometric synthesis, its basic innovation was a shift away from the promotion of alternative observation-based visualizations (= ordinations) of geometric variation among objects to an agreement that the primary task of morphometrics should be to make whole landmark configurations into "variables" that could then be used to achieve ordinations of increased generality (Bookstein 1993). Work since the geometric morphometric synthesis has largely focused on applications of landmark-based morphometrics in a variety of practical contexts (Marcus et al. 1993; Marcus et al. 1996) and the interpretation of outline data within geometric morphometric theory-practice (e.g., Bookstein and Green 1993; Bookstein 1996a,b, 1997; MacLeod 1999, in press a). Although much methodological research remains to be done, there presently exist tools sufficient for the analysis of landmarks, outlines, and any conceivable combination of these to data categories within a unified data analytic framework that supports the extensive and elegant geometric presentation of analytic results.

As an example of the power of the new geometric morphometrics this contribution looks back upon several previous qualitative and semi-quantitative attempts to derive shape-based classification schemes for a variety of geological objects. The purpose of these revisitations is to 1) assess their accuracy with a formally geometric context, 2) employ the modeling tools of geometric morphometrics to devise new and more accurate/representative depictions of shape index classes for descriptive geological use, and 3) to review methods that can be used to more objectively assign objects to shape classes, however defined or constructed. In one sense this investigation can be seen as a simple demonstration of a geometric morphometric technique in a novel—and somewhat antiquarian—context. After all, if one has the power to geometrically represent shapes and morphometrically study their patterns of variation with rigor and sophistication then the need for *a priori* shape classifications diminishes. The results of such morphometrical investigations by themselves should enable investigators to characterize the pattern(s) of variation and determine whether subordinate groupings indicative of class-level distinctions are present regardless of whether standard semi-quantitative classifications exist. In a deeper sense, though, the abstract shape-classification schemes have become so well-integrated into the corpus of geological practice and proven so useful for organizing geological observations, that it seems churlish to advocate their abandonment at this juncture. Nevertheless, it also seems ill-advised to allow incorrect, non-representative, or misleading indices, scales, exemplars, etc. to persist in the geological literature simply for tradition's sake.

The Method

The method I will employ for this investigation is eigenshape analysis (Lohmann 1983; Lohmann and Schweitzer 1990; MacLeod 1999). In its current formulation (MacLeod 1999) eigenshape analysis can be viewed as a special case of relative warp analysis (Bookstein 1991) that is defined over an extraordinary variety data types (e.g., landmarks, boundary coordinates, mixtures of both, angle-based shape functions) and supports a wide variety of graphical data-result representation strategies (e.g., multivariate ordinations, biplots, deformation diagrams, thin-plate splines, see also MacLeod in press a).

Eigenshape begins—as do all geometric morphometric procedures—with the collection of landmarks or semi-landmarks for all specimens within a sample. These two or three-dimensional coordinate positions should correspond to one another across the sample, though the nature of the correspondence may vary. For example, Figure 1 illustrates two different ways of representing the mammalian distal phalanx in lateral view are shown. Both methods are compatible with eigenshape analysis.

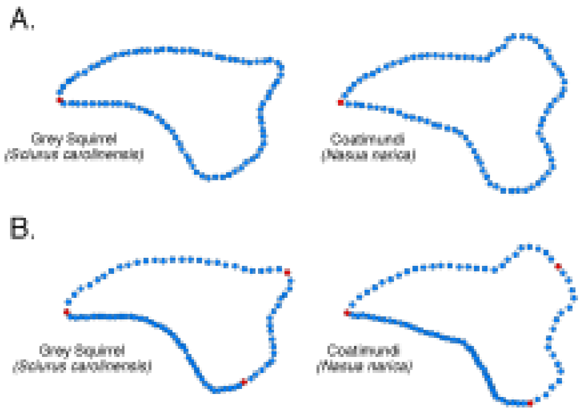

Figure 1. Alternative form digitzation (= sampling) schemes compatible with eigenshape analysis. A. Standard eigenshape analysis digitzation scheme. Red coordinate represents the starting point of for shape function conversion. This point may be selected by the user or algorithmically determined by successive rotation of forms and comparison to a reference shape. B. Extended eigenshape analysis digitization scheme. Red coordinates representing positions of landmarks used to register the forms with one another. Note that the number of semi-landmarks (= boundary coordinates) differs between outline segments. This results from the fact that some regions of the boundary contain more shape information than others, and so require more semi-landmarks to represent to a consistent level of resolution. It is important to note that in both digitization schemes the semi-landmark sequences are constrained to correspond to one another as if they were landmarks. This is the result of regarding the inter-landmark outline segments are being geometrically homologous to one another and representing that sequence by an arbitrary sampling convention.

Once the shapes of the sample have been represented by coordinate-point digitization the coordinate values are transformed into a shape functions (e.g., the ϕ or ϕ* functions of Zahn and Roskies 1972, the tangent angle function of Bookstein 1978, a column vector of raw coordinate values), assembled into a data matrix, and submitted to covariance-based singular-value decomposition (SVD). The resulting eigenvector array summarizes sample-optimised, orthogonal, shape-variation contrasts that can be used as the latent axes of an ordination system or transformed back into their equivalent geometric representations for inspection and comparison (see Schweitzer and Lohmann 1990; MacLeod and Rose 1993; MacLeod 1999, in press a for details).

In addition to the ordination capabilities of eigenshape analysis, the ability of the technique to move freely between the ordinative-analytic and geometric-representational domains means not only that very explicit shape-based hypotheses can be examined and tested, but also that the procedure supports interactive shape modelling-morphing within a highly specified, deterministic and quantitative framework. This, in turn, provides eigenshape analysis with a greatly enhanced range shape analysis capabilities. The present contribution focuses on the modelling-morphing utility of eigenshape analysis as a generalized tool for creating accurate and useful generalized shape classification systems whereas previous articles have focused on its ordination and hypothesis-testing capabilities (see references above).

The Analysis of Simple Closed Curves

One of the most common geological shape classification systems is the 'Powers Roundness Scale' (Powers 1953) for sedimentary particle shape (Fig. 2). Friedman and Sanders (1978) listed eight factors that determined the shape of sedimentary particles and there is an extensive literature concerned with the use of particle shape to infer a wide variety of physio-historical properties of sedimentary rocks. Like most such scales, the Powers scale makes a distinction between particle roundness or sphericity and angularity. This reflects a geometric distinction between the overall shape of the particle and the texture of its surface that was first described by Wadell (1932, 1933, 1935).

For most geologists—since instruction in the sphericity-roundness scale part of all geologists' introduction to the science—sphericity is conceptually defined as the degree to which a particle approximates the shape of a sphere and operationally defined as the ratio between the surface area of the particle and the surface area of a sphere having equal volume. Since volumes are difficult to measure accurately. Most commonly applied equation for estimating sphericity is that of Sneed and Folk (1958) for maximum-projection sphericity (Ψ_p).

$$\psi_p = \sqrt[3]{\frac{S^2}{LI}}$$

Where: L = major (long) axis of the bounding ellipsoid.
I = intermediate axis of the bounding ellipsoid.
S = minor (short) axis of the bounding ellipsoid.

However, even these empirical parameters are difficult to measure on small particles. In these instances the most commonly used method of estimating the sphericity of a sample is to inspect the particles by eye and mentally compare the particle shapes to published scales of which the Powers scale is one example. [Note: while the morphometric estimation of sedimentary particle shape represents a prolific research program in its own right (e.g., Ehrlich et al. 1980; Boon et al. 1982; Kennedy and Ehrlich 1985) these studies constitute a specialized branch of descriptive sedimentology whereas the qualitatively assessed estimates of particle shape (sphericity and roundness) represent routinely reported basic data for virtually all types of sedimentological and engineering analysis.]

Wadell's (1932) concept of roundness is even more complex and difficult to quantify than that of sphericity. Roundness is a measure of the degree of sharpness or curvature of a particle's edges or corners. Computationally this translates (in two dimensions) into the ratio between the average radius or the particle's corners and edges and the radius of the maximum inscribed circle. Wadell (1935) used a 'circle scale' (= set of concentric circles of known radii)

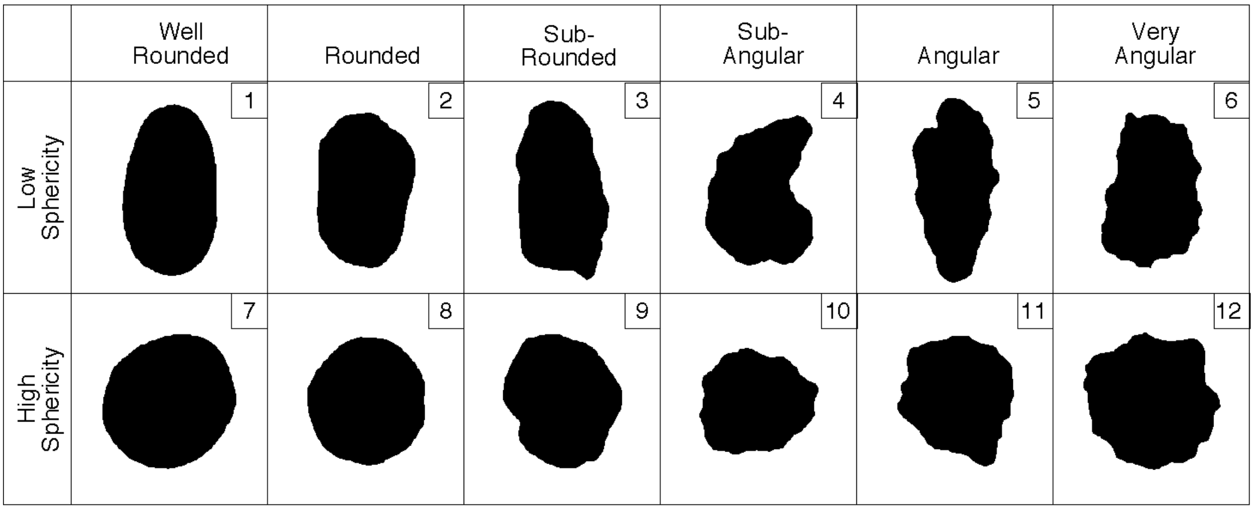

Figure 2. The Powers (1953) sedimentary particle roundness (= sphericity) and angularity scale. These outlines were originally created from drawings of actual sedimentary particles after determining their sphericity and angularity according to the methods outlined by Wadell (1935). Numbers in the upper right-hand corner of the scale cells are arbitrary identification numbers that will be used in subsequent ordination plots.

and a camera lucida to create his scale. However, it is evident from his figure illustrating this procedure (Wadell 1935, Fig. 2; see also Krumbein 1940, Fig. 11; Krumbein and Sloss 1963, Fig. 4-10) that the definition of particle 'corners' and 'edges' was decidedly subjective with certain regions of the form being measured much more intensively than others. Moreover, it is widely acknowledged that such a measurement strategy is inappropriately time-consuming for routine analysis. As a result, Wadell (1933), Krumbein (1941), Powers (1953), and others formulated sphericity-roundness classification schemes in which drawings of exemplar shapes (based on real sand grains) represented 'type' class members. These scales are reproduced in essentially every introductory sedimentology text and geological field guide for use in classifying sediment samples with the Powers scale being among the most popular.

While there is nothing wrong with the class-exemplar approach per se—especially for rapid, first-approximation studies—the utility of these scales suffers from there being either insufficient or over-abundant information on intra-class variability (see Powers 1953). These criticisms are inherent in the static nature of the diagrams representing the classification and the use of only a single grain as a class exemplar (but see Krumbein 1941 for an interesting counter-example of the latter). Regardless, the use of real grains as exemplars for idealized geometric concepts can also be criticized on the utilitarian grounds that it fosters confusion between inter- particle differences that are independent from either sphericity or roundness aspects of shape variation and those that are wholly dependent on them. In a sense, these idealized exemplar systems are sub-optimal because they contain too much information, much of it extraneous to the representation of object sphericity and roundness.

The existence of this extraneous information can be easily revealed via eigenshape analysis of the Powers (1953) exemplar set. The twelve idealized shapes represented in the Powers scale were digitized at a resolution of 200 equally spaces points about the two-dimensional periphery. These boundary coordinate datasets were then interpolated to a consistent minimum boundary-representation ($\geq$ 1.0 percent of total perimeter) across the sample (see MacLeod 1999). This operation had the effect of reducing the number of points needed to represent the boundary from 200 to 180. While this is not a marked reduction in dataset size, the point of this operation is to ensure adequacy of resolution. The x,y boundary coordinates were then transformed into their equivalent ϕ shape functions (Zahn and Roskies 1972) and iteratively adjusted to achieve consistent and maximally congruent shape orientations. The subsequent SVD analysis of the pairwise covariance matrix calculated from these shape functions resulted in the specification of a series of eigenvectors (= eigenshapes). The lengths of these 'eigenshape vectors' are proportional to the amount of observed shape variance subsumed by the axis while their orientations constitute a set of orthogonal shape trends that represent the most common mode of shape variation across the entire sample (Eigenshape 1, or ES-1) and the most important shape difference trends between various subgroupings (ES-2 – ES-n). As with most eigenanalysis-based procedures, these eigenshape vectors can be used to define the axes of a 'shape space' within which the original shapes may be ordinated. Inter-object proximity within such a space represents degrees of covariance-based similarity between shapes and the latent shape trend represented by the eigenshape axes.

Scatterplots of the common shape trend (ES-1) and the two most important intra-sample shape difference trends (ES-2 and ES-3, Fig. 3) shows that the structure of the shape information—as assessed by eigenshape analysis—is considerably more complex than suggested by Powers' (1953) simple sphericity

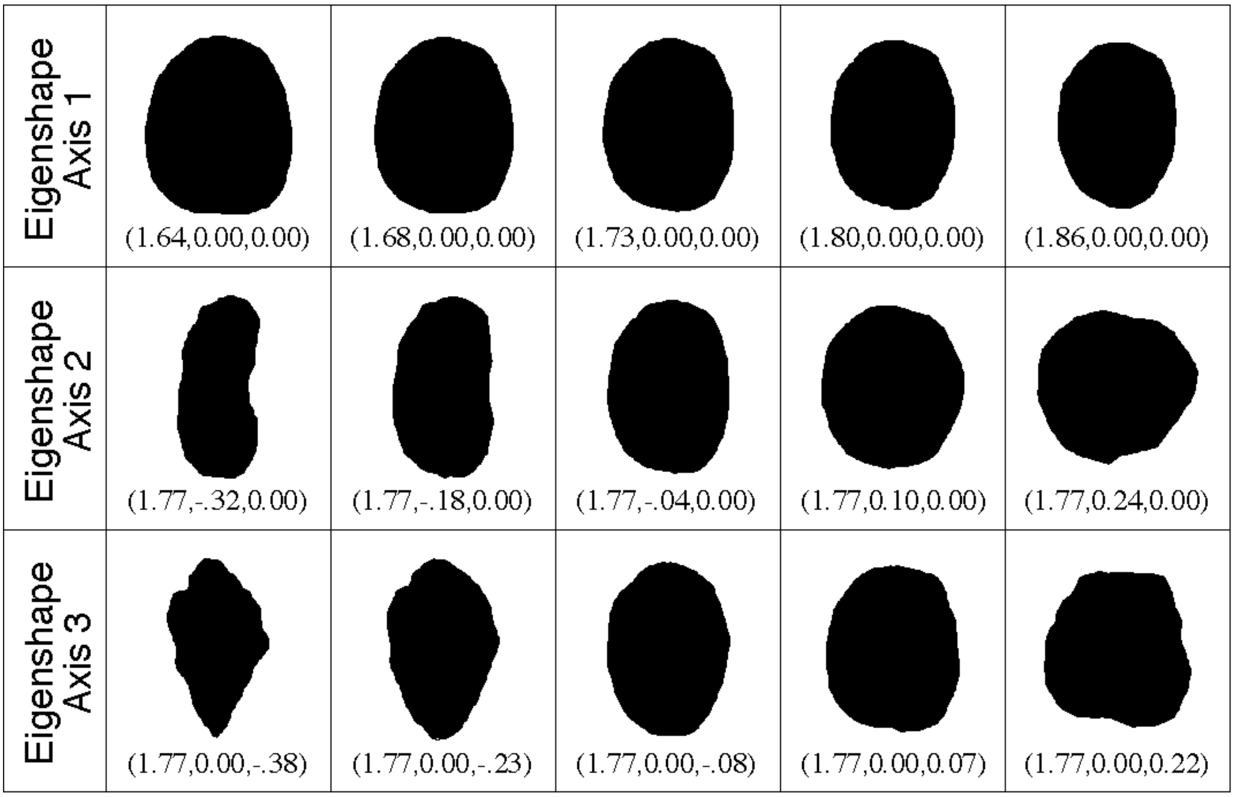

Figure 3. Shape models derived from the first three eigenshape axes of a covariance-based standard eigenshape analysis of the Powers (1953) exemplar shapes. Numbers at the bottom of each model cell represent coordinate positions of the model within the shape space defined by these three axes. See text for discussion.

and roundness classification system. Distinctions between these objects in terms of the classical sphericity index are represented by the eigenshape results, but these are partitioned between two different generalized shape trends rather than a single, unified shape factor. In addition, the complex arrangement of shape subgroupings present on these axes suggests that shapes 4, 5, and possibly 6 represent substantially different shape types than those constituting the main shape groupings. To be sure, these are the most angular shapes in the sample. However, the fact that they do not form a unified group among themselves is troubling in the sense that the Powers scale suggests that they should be more similar to each other than to any other shapes in the sample. These eigenshape results do not support a traditional interpretation of the Powers (1953) exemplar shapes.

The underlying geometric explanation for the ordinations shown in Figure 3 can be revealed by modeling the eigenshape axes across the range of covariance values represented by the sample. Figure 4 illustrates the results of this modeling exercise for a series of five positions along the first three eigenshape axes. These models show that aspects of the traditional low-sphericity to high-sphericity shape contrast are part of the overall shape similarity (ES-1) *and* shape difference (ES-2) trends exhibited by these exemplars rather than being confined to a single, independent shape-variation factor (as implied by the Powers classification). Interestingly, the third most-important shape difference trend would not be interpreted as a texture-based angularity factor, but rather as another whole-particle shape change factor with the predominant geometrical contrast residing in a difference between teardrop-shaped and ellipsoidal particles. As with the ordinations shown in Figure 3, these geometric model-based results suggest that the Powers scale is a somewhat poor and decidedly idiosyncratic way of structuring the shape information present in these exemplar outlines.

While eigenshape analysis has provided an alternative geometry-based assessment of the primary shape variation trends within the Powers (1953) exemplar shape sample, it can also be used to go beyond assessment into creation of a more consistent

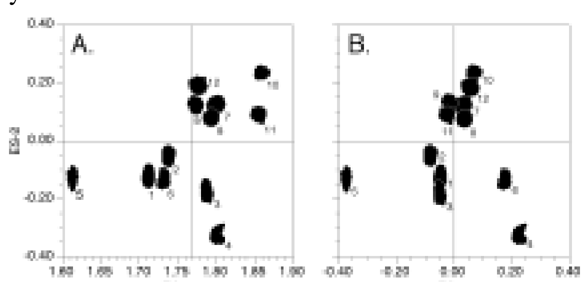

Figure 4. Scatterplots of the Powers (1953) sedimentary particle shapes within two shape planes formed by A. eigenshape axis 1 (ES-1, λ = 96.57%) and eigenshape axis 2 (ES-2, λ= 0.83%) and B. eigenshape axis 2 (ES-2) and eigenshape axis 3 (ES-3, λ = 0.63%). Small numbers beside object icons refer to the exemplar identification numbers in Figure 3. Horizontal and vertical lines represent the trajectories of the eigenshape model series shown in Figure 3.

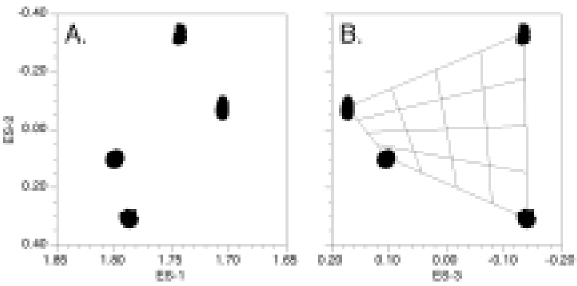

Figure 5. Scatterplots of the Powers (1953) sedimentary particle end member shapes (= shapes 1, 6, 7, and 12 in Figure 2) within two shape planes formed by A. eigenshape axis 1 (ES-1, λ = 97.34%) and eigenshape axis 2 (ES-2, λ = 1.73%) and B. eigenshape axis 2 (ES-2) and eigenshape axis 3 (ES-3, λ = 0.63%). The rectilinear coordinate system drawn between the end member shapes in B. represents the locations of intermediate shape models shown in Figure 6.

and representative exemplar shape classification system. The problem with the Powers (1953) exemplar set is that it consists of drawings of real sediment particles that have been selected to illustrate only certain aspects of their combined shape variation. Users of such a classification have difficulty separating the sphericity and roundness factors from other aspects of shape variation because the human visual system is not designed to segment and analyze images in this way (Gonzalez and Wintz 1977). Nevertheless, the geometric-analytic approach represented by eigenshape analysis can be combined with the end-member shapes of the Powers (1953) scheme to produce a more consistent and useful result.

Figure 5 shows the ordination-level results of an eigenshape analysis using the four end-member shape of the Powers (1953) scheme (shapes 1, 6, 7, and 12 of Fig. 3). These end-member shapes define a rough quadrilateral on the plane formed by the two most important shape difference axes (ES-2 vs. ES-3, Fig. 5B). [Note: the (arbitrarily ordered) scales defining these axes have been reversed in order to bring the resulting ordination into conformance with the standard representation of the Powers (1953) classification.] By using these end-member shapes to define a rectilinear deformation matrix it is possible to locate a series of nodes that represent all possible linear geometric transitions between the end-member shapes. Once these coordinate locations have been defined it is an easy matter to use the shape modeling capabilities of eigenshape to represent those transitional states in a manner that qualitatively captures the distinctions between roundness and sphericity intended by Powers (1953, and by Wadell 1935, and Krumbein 1941 for their exemplar-based classification series) while at the same time minimizing the influence of extraneous shape variation patterns. The resultant exemplar set (Fig. 6) represents a dynamic compromise between actual particles (= the four corner shapes) and the conceptual transitions that uses geometry to link the real world of sedimentology and particle analysis to the abstract concepts of sphericity and roundness in a rigorous yet useful way. Moreover, because this shape deformation system is defined

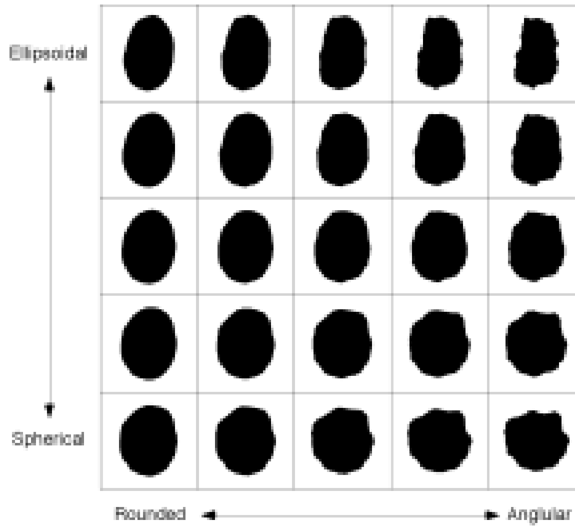

Figure 6. Alternative, comparative, sedimentary particle roundness-angularity and spherical-ellipsoidal shape classification based on eigenshape analysis results. Corner shapes are end-members of the Powers (1953) sedimentary particle shape classification. This scale eliminates most of the extraneous shape variation within the Powers (1953) exemplars. [Note: since end-member reference shapes do contain some amount of extraneous shape variation this source of error has not been completely eliminated.] A 'pure' classification could be achieved by using geometric methods to produce ideal hypothetical shapes based on the Wadell (1935) criteria and repeating the eigenshape analysis. Because this scale is based on a mathematical model sets of hypothetical intermediate shapes at any location within the shape space can be calculated. In addition, the digitized shapes of actual grains can also be projected into this space to determine their precise positions.

geometrically, analogous procedures can be used to reconstruct exemplar shapes for any position with the deformation space. The resultant figures can then be arranged or otherwise portrayed in any manner that the user deems suitable for the task at hand. Of course, the set of eigenshapes that define this space can also be used to project external shapes onto this shape difference plane in order to facilitate quantitative comparisons between individual particle shapes and the eigenshape-modelled sphericity-roundness scale (see MacLeod and Rose 1993).

The Analysis of Closed Curves with Landmarks

While the class of geometric analyses represented by closed curves such as individual sedimentary particle shapes encompasses a wide range of geological form-classification systems, it by no means exhausts the topic or the analytic-modeling capabilities of eigenshape analysis. In many instances geologists wish to analyze closed curves and to take advantage of point locations along the shape boundary that correspond to one another across the shape sample. Jack Wolfe's leaf shape classification system for paleoenvironmental analysis (Wolfe, 1993, 1995) contains several outstanding examples of this situation in the form of his leaf margin characters.

The main thrust of Wolfe's work was to establish a series of physical (or physiognomic) leaf characteristics that could be scored qualitatively and the leaf

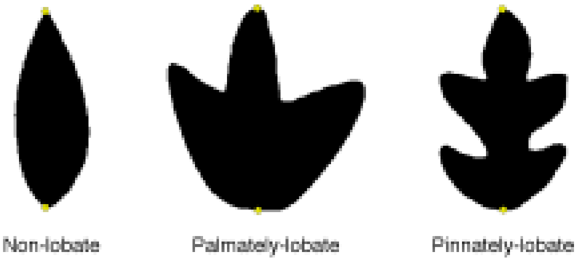

Figure 7. Exemplar shapes of leaf-margin character states used by Wolfe (1993, 1995) in climatological analysis. Note positions of landmarks used by extended eigenshape analysis (MacLeod 1999) to subdivide the leaf margins into two segments. Redrawn from Wolfe (1993).

assemblage scores for a local flora used to infer past climates via input into a transfer-function type multivariate analysis (e.g., correspondence analysis, canonical correspondence analysis) that had been calibrated using modern floras and climatic data. All of the 29 characters Wolfe uses in his current classification pertain either to the leaf margin or leaf size and all have been illustrated as end-member states with text-based definitions of class boundaries (Wolfe 1993). Figure 7 illustrates the end-member states representing the 'lobed' leaf shape character whose definition was given as follows.

"A species receives a score of 0 if no leaves are lobed, a score of 0.5 if some leaves are lobed and some are unlobed. and a score of 1.0 if all leaves are lobed. Lobing can be either pinnately lobed (for example, many Quercus) or palmately lobed (for example, many Acer). Both pinnately and palmately lobed [leaves] were scored separately but were later combined because separate scoring appeared to produce no refinement. In order to be pinnately lobed, a lamina must be incised so that a line connecting the sinuses between the lobes is approximately parallel to the midrib. …In palmately-lobed leaves, the lobes are entered by a primary vein that originates near the base of the leaf." (Wolfe 1993, p. 21).

While on first reading this seems like a reasonable and objective subdivision of the possible morphological variation types, closer inspection shows that it focuses on the distinction between pinnate and palmate lobe types—a distinction that is not used in scoring the character—while allowing the class boundary between 'lobed' and 'unlobed' states to remain undefined. This oversight conflicts with Wolfe's stated desire (1993, 1995) that the leaf margin character states analyzed be defined rigorously. For user's of Wolfe's scheme (some of whom may not be paleobotanists) the most practical classification would be graphical scheme that illustrates the range of acceptable intra-class variational types, perhaps augmented by a textual description of the class boundary criteria. While the latter is a task for paleobotanists, the former can be addressed by a geometric morphometrician.

From a morphometric point-of-view, the leaf-lobe character represents a more complex problem that the sedimentary particle shape example. While the leaf margins are free to assume a wide variety of shapes, there are two fixed points on the figure that may be considered geometrically homologous across most leaf samples. These are the 'landmarks' of the leaf base and the leaf tip. Accordingly, any method of

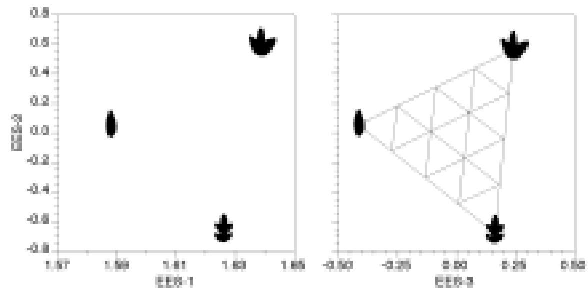

Figure 8. Scatterplots of the Wolfe (1993) leaf margin lobateness exemplars within the two shape planes formed by A. extended eigenshape axis 1 (EES-1, λ = 88.43%) and extended eigenshape axis 2 (EES-2, λ= 8.72%) and B. extended eigenshape axis 2 (EES-2) and extended eigenshape axis 3 (EES-3, λ = 2.85%). The ternary coordinate system drawn between the end member shapes in B. represents the locations of intermediate shape models shown in Figure 9.

geometrical analysis that attempts to represent these data should incorporate these distinctions into the analysis and ensure that shape functions are appropriately constrained (by the fixed landmark points) and matched (left margin with left margin, right margin with right margin) during its course.

The eigenshape approach in its extended form (MacLeod 1999) fulfills these criteria. Instead of matching shape functions across the entire perimeter (as was done in the sedimentary particle example), extended eigenshape analysis breaks the shape function into two subfunctions at the landmark points and ensures that boundary coordinates from (say) the left leaf margin are only matched with ordinally corresponding points from the left margins of other leaves during the calculation of the pairwise covariance matrix. This landmark point-based registration of the objects in a sample can be extended to any number of landmarks so long as they are located on the object perimeter and has the effect of localizing shape differences that otherwise (e.g., under the standard eigenshape approach) would be spread over much broader areas of the form due to a relative lack of tight geometric correspondence between shape functions. [Note: this relative lack of correspondence is due to the lower information content of landmark-free outlines; see MacLeod 1999 for a more complete discussion and additional examples.]

Application of extended eigenshape analysis to the Wolfe (1993) leaf lobateness character exemplars (Fig. 8) shows that non-lobate, pinnately lobate, and palmately lobate leaves form an approximately equilateral, triangular distribution in the shape space formed by the three eigenshape axes. This suggest that despite Wolfe's (1993) ambiguous results regarding climatological distinctions between pinnately lobate, and palmately lobate leaves, from a geometric point-of-view pinnately lobate leaves and palmately lobate leaves are as distinct from one another as either are from non-lobate leaves. Note also that the ordinations of these end-members in the shape-difference space (Fig. 8B) is such that neither the critical distinction between non-lobate and pinnately lobate leaves, nor the similarly critical distinction between non-lobate and palmately lobate leaves, is captured by any of the eigenshape axes alone.

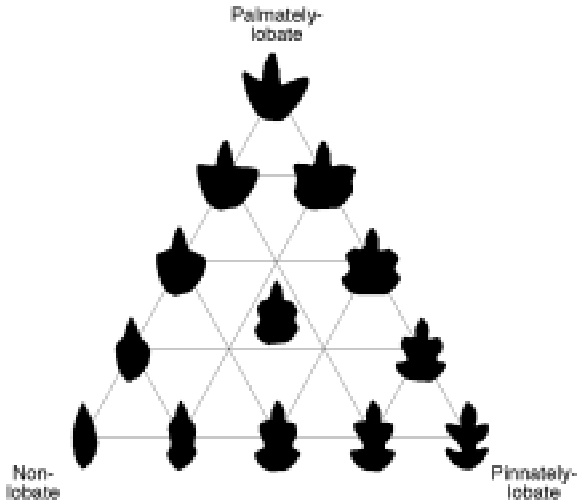

Figure 9. Alternative, comparative, leaf margin lobateness shape classification based on extended eigenshape analysis results. Corner shapes are exemplars of the Wolfe (1993) classification. This scale illustrates the range of intermediate shapes consistent with Wolfe's (1993) a priori-defined exemplars. See text for discussion

As with the sedimentary particle analysis, the modeling capabilities of eigenshape analysis—which are retained in this method's extended form—can be used to devise a qualitative leaf-lobateness scale for use by investigators wishing the classify their leaves in accordance with Wolfe's (1993) scoring scheme. By explicitly illustrating forms of variation consistent with the various lobateness concepts such a scale would be of arguably greater utility (especially for non-paleobotanists or paleobotantists in remote locations) than the end-member exemplars and written descriptions provided by Wolfe (1993). A ternary classification system seems appropriate for this purpose. Figure 9 shows the results of this modeling exercise.

Once again, the semi-quantitative shape classification system construction via geometric morphometric analysis of end-member exemplars exhibits many heuristic advantages over the simple illustration of the end-members or the illustration of intra-class patterns of variation with real species that vary in ways extraneous to the relevant classification criteria. Over this range of models Wolfe's (1993) classic non-lobate leaf morphology can be seen as being restricted to a relatively small area of the available shape space. Surprisingly, though, is the fact that the more non-lobate members of the non-lobate – palmately lobate axis exhibit physiognomic morphologies that would be placed in completely different characters within the overall Wolfe (1993) classification. In particular, both the median and submedian models along this axis exhibit the attenuated apices (= drip tips) that Wolfe would code under a separate apex character (see Wolfe 1993, p. 25, Fig. 8R and 8S). Similarly, the median model also exhibits the maximal width in the topmost third of the periphery, a physiognomy consistent with Wolfe's leaf shape: obovate character state (see Wolfe 1993, p. 25, Fig. 8AA or Wolfe 1995, p. 123, Fig. 1C). These unexpected correspondences with other physiognomic characteristics within the putatively distinct 'lobed' leaf character underscore the hidden geometric complexities inherent in formulating independent shape characters from end-member exemplars in the absence of geometric-morphometric analysis, as well as highlighting the advantages of adopting this more comprehensive and information-rich approach.

The Analysis of Open Curves

Several of Wolfe's (1993, 1995) other qualitative shape classifications offer good examples of the last major group of shape classification problems, the

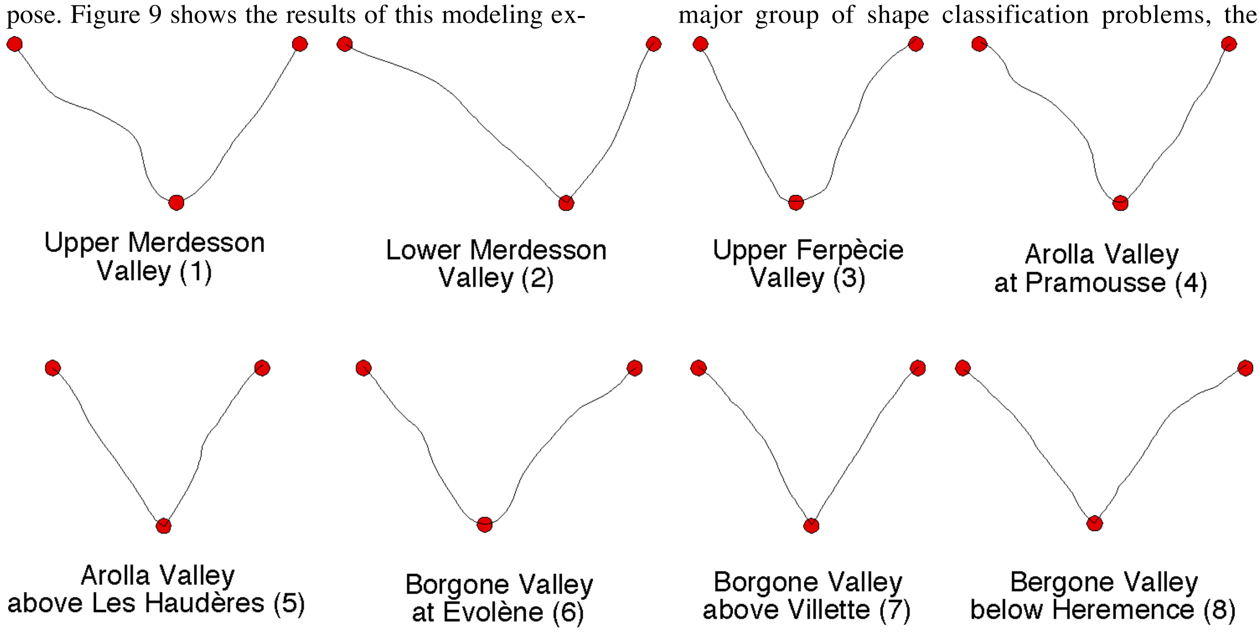

Figure 10. Exemplars of Small's (1972) alpine valley profile shapes used for a discussion of geomorphological landforms characteristic of glaciated terrains. Note positions of landmarks used by extended eigenshape analysis (MacLeod 1999) to subdivide the valley profiles into two segments. Numbers following valley-profile location names are arbitrary identification numbers that will be used in subsequent ordination plots. Redrawn from Small (1972).

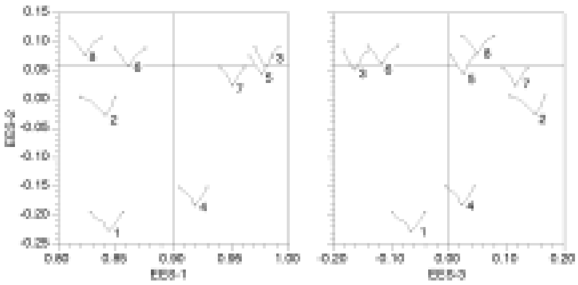

Figure 11. Scatterplots of the Small (1972) alpine valley profile exemplars within the two shape planes formed by A. extended eigenshape axis 1 (EES-1, λ = 95.35%) and extended eigenshape axis 2 (EES -2, λ = 1.38%) and B. extended eigenshape axis 2 (EES -2) and extended eigenshape axis 3 (EES -3, λ = 1.22%). Small numbers beside object icons refer to the exemplar identification numbers in Figure 10. Horizontal and vertical lines represent the trajectories of the eigenshape model series shown in Figure 12.

quantification and analysis of open curves either with or without intermediate landmarks. However, in the interest of diversity I will leave these to another time (or another investigator) and turn to another, completely different earth science discipline, that of geomorphology. As with much of descriptive sedimentology, and virtually all of paleontology, geomorphology is a comparative science. The recent fashion for investigations that make use of geographical information systems (GIS) notwithstanding, there remain relatively few examples of geographical morphometrical analysis despite the fact that qualitative classifications for landform features abound in geographical and geological textbooks.

Take, for example, the conceptually simple distinction between a V-shaped and a U-shaped valley. The former is typically regarded as typically being formed by fluvial processes whereas the latter is typically interpreted as evidence for glaciation at some time in the past. Small (1972) provides a series of valley profiles from the glaciated Val d'Hérens in Valais, Switzerland that suggest a more complex interpretation for such profiles.

The eight profiles provided by Small (1972, reproduced in Figure 10) were taken on a roughly south-north traverse through the complex of valleys comprising the Val d'Hérens, Switzerland. Of particular note in this complex (but existing in many other Alpine valleys with evidence of extensive glaciation) are several sections exhibiting the V-shaped profile thought to be more characteristic of fluvial erosion. Small (1972) acknowledges the discrepancy, but challenges previous explanations for the existence of these V-shaped valleys as representing postglacial deepening of these sections by fluvial processes. In defense of his position Small (1972, p. 358-359) argues that "Not only has the time-laps since deglaciation been quite insufficient, but the V-sections are often occupied by masses of valley-side moraine. It can only be inferred that, for some reason, ice passed through these V-profiles in large quantities without altering their form, whereas in other places (as between Les Haudères and Evolène) the more normal U-profile was developed."

Any hypothetico-deductive test of generative hypotheses that might account for these observations and resolve the controversy would need to devise a way to classify valley shape profiles. Of course, one could simply subdivide the profiles according to whether the tangents in the vicinity of their nadirs formed one broadly variable group or two distinct groups of orientations. However, this classification would focus entirely on a restricted region of the valley profile and may well ignore crucial evidence along the valley walls (e.g., modifications of the profile symmetry that may covary with substrate type indicating structural control of the valley morphology.

Extended eigenshape analysis is as well-suited to the analysis of open curve geometrical problems as to close-curve analyses. For this particular example, in addition to the two end-points of the open curve, it would be advantageous to adopt a practical definition of the valley nadir (e.g., lowest elevation along the profile) and represent that point by a landmark. This operation can be justified on the grounds that this point is there the force of fluvial activity would likely be concentrated if it were an important factor in shaping the valley and it would ensure a better registration of the curves than if they were registered only by their endpoints. Extended eigenshape analysis would proceed in a manner identical to that described for the leaf margin analysis (above). Results of the ordination of Small's Val d'Hérens profiles within the shape space formed by the first three eigenshape axes are shown in Figure 11.

Interestingly, these results show that the dominant geometrical contrast exhibited by these profiles does not lie along a simple U-shape (profiles 1, 3, 4, 6) vs. V-shaped (profiles 2, 5, 8, 7) axis. Rather, the contrast that accounts for the greatest proportion of observed shape difference (represented by extended eigenshape axis 2, EES-2) is that between the roughly symmetrical profiles (2, 3, 5, 6, 7, 8) and the strongly asymmetrical profiles (1, 4). The fact that both of the asymmetrical profiles are very similar to one another, but located in different regions of the Val d'Hérens complex (and so are very different sizes) may be suggestive of structural control. However, a larger sample size and information about outcrop patterns would be needed to confirm this hypothesis. The expected U–V contrast is represented on EES-3 where a pronounced gap exists between the symmetrical U-shaped profiles (3, 6) and the symmetrical V-shaped profiles (2, 5, 7, 8). This gap may also be indicative of structural control if it could be shown that the clear contrast between the symmetrical profiles captured along ES3 covaries with some aspect(s) of the underlying geology.

The shape modeling procedures outline above can be applied to create a series of semi-quantitative profile models for use in assessing the predominant modes of shape variation in these data and classifying other valley profiles (Fig. 12). Along-axis model for the first extended eigenshape axis (modeled through the main scatter of shapes at EES-2 = 0.60, see Fig. 11) suggest that the predominant shape similarity axis

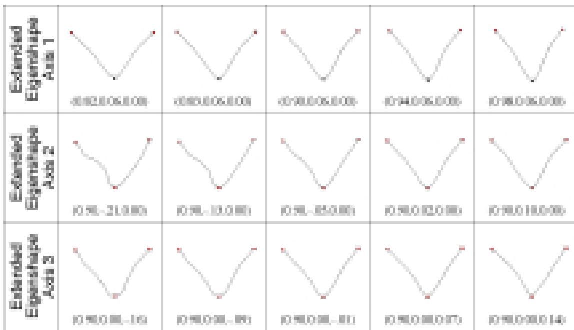

Figure 12. Shape models derived from the first three extended eigenshape axes of a covariance-based extended eigenshape analysis of the Small (1972) exemplar shapes. Numbers at the bottom of each model cell represent coordinate positions of the model within the shape space defined by these three axes. See text for discussion.

represents a contrast between slightly more open (low values) and slightly more closed (high values) V-shaped profiles.. There is some rounding of the valley nadir present on this axis, but it is a decided minor component of the overall shape variation. The second extended eigenshape axis of these data represents suggests that the most important shape dissimilarity trend involves a change in the profile asymmetry with low values representing the presents of kinks in the left-hand portion of the trace with a largely unkinked right-hand trace, and high values representing kinks in the right-hand trace with an unkinked left-hand trace. The classic U vs. V profile distinction is best expressed along the third extended eigenshape axis. Overall these data suggest that the U–V profile contrast represents a inferior mode of shape dissimilarity relative to that of profile symmetry.

Discussion

Relation of eigenshape analysis to other geometric morphometric methods.—It is noteworthy that eigenshape analysis was used to address the diversity of morphological problems considered above. While other approaches to morphometrics or shape modeling could be used to address some of these situations, few, if any, could perform across the entire range with as much facility in their traditional guises. Does the mean that eigenshape is a uniquely generalized or useful approach to morphometric analysis and shape modelling? My experience with the broad range of contemporary morphometric procedures suggests that the answer to this question is “No.” Rather, previously commented upon differences between eigenshape and other morphometric procedures, as well as between ‘coordinate methods’ (e.g., Procrustes-registered warp analysis) and ‘non-coordinate-methods’ (e.g., eigenshape analysis, fourier analysis) have arisen because of an incorrect understanding of the relations between these methods, appropriate comparisons, and the nature of the ‘morphometric synthesis’. A more detailed understanding of these topics extends that synthesis and suggests additional tools that geologists might use in their investigations of geological forms.

Bookstein (1991) explained the rationale behind the term “geometric” morphometrics as stemming from a desire to preserve the original measurement space of landmarks that previous morphometric data analysis procedures (e.g., distance-based multivariate morphometrics and outline-based methods like Fourier analysis and eigenshape) had either failed to achieve or recognise as necessary. This was because it was thought that these latter methods represented transformations of the observed data into an abstract variable space that was incommensurate with the Euclidean space of the original observations. Once the original data had been transformed into this latent variable space it was thought that no meaningful return to the measurement space was possible because the tolopogical information required to reconstruct the morphologies had been lost. However, as the example analyses above illustrate (see also MacLeod and Rose 1993; MacLeod 1999, in press a) the eigenshape method produces results that are fully convertible between the abstract space of eigenanalysis-optimised coordinate systems and the Euclidean space of the original measurements. This counter-intuitive result was possible because there is no fundamental difference between the current spectrum of geometric morphometric data-analysis methods and older, coordinate-based multivariate methods such as eigenshape or fourier analysis.

Like all morphometric methods, eigenshape begins with the representation of an object’s shape as a shape function: a linear combination of geometric observations. While eigenshape analysis traditionally uses the Zahn and Roskies (1972) ϕ function, any shape function could, in principle, form the basis of an eigenshape analysis (MacLeod 1999). This part of an eigenshape analysis is analogous to the representation of an object’s shape by radial or elliptical fourier decomposition (see Lestrel 1997), by the arrangement of a series of *x,y* coordinate located into a matrix of column vectors (see Bookstein 1991, MacLeod, in press a, b), or indeed by the expression of shape as a matrix of inter-landmark distances. All are shape functions and all can serve as the basis for an eigenshape analysis. By the same token, all of these coordinate-referenced shape functions should, in principle, be able to serve as the basis for other types of geometric morphometric data analysis procedures. Inter-landmark data differ from coordinate-referenced shape functions in this regard because they fail to preserve information on relative spatial location over the form. Nevertheless, the reliance on the quantification of shape by means of a linear function represents a level of deep similarity among all morphometric data-analysis techniques.

It is important to understand that this underlying similarity among geometric morphometric methods transcends the older and somewhat misleading distinctions that were previously drawn between ‘landmark-based’ and ‘outline-based’ methods (e.g., Bookstein et al. 1982, Read and Lestrel 1986). All morphometric measurements are based on landmarks. Landmarks are defined operationally as relocatable coordinate positions on an object or image in a two-dimensional or three-dimensional Euclidean measurement space. This definition is identical in concept to—though more specific than—to that offered by Bookstein (1991, p. 2). The Euclidean distances of multivariate morphometrics (e.g., Blackith and Reyment 1971; Reyment et al. 1984) are distances between landmarks; he boundary coordinates of Fourier (e.g., Lestrel 1997) and eigenshape (e.g., MacLeod 1999) analysis are landmarks; and the landmarks of geometric morphometrics (e.g., Bookstein 1991) are landmarks.

Bookstein (1991) identified three classes of landmarks: discrete juxtapositions of structures (Type 1), maxima of curvature (Type 2), or extrema (Type 3). This classification focuses attention on the type of information necessary to identify or relocate each landmark. Type 1 landmarks require the most information to identify and may occur at any point on or within a form so long as that form is composed of different structures. While these landmarks are constrained to exist on the boundaries (= outlines) of these structural components or tissue-defined regions, their locations are not determined by any characteristics of the overall boundary or outline. Type 2 landmarks lie on the boundaries of single structures or regions and are defined by the nature of the curving surface of that boundary. This definition constrains Type 2 landmarks to be located relative to the distribution of adjacent boundary coordinates. Type 3 landmarks represent those coordinate locations on single structures (irrespective of whether the structure is composed of various substructures or regions) that represent the extremes of the structure's boundaries. Like Type 2 landmarks these points are constrained to lie on the object's periphery or outline.

No consideration has been traditionally given to the nature of any substructure when locating Type 3 landmarks. Their definition is dependent entirely on the nature of the outline (= by the distribution of adjacent boundary coordinates), upon the orientation of the object, and on the number of axes one wishes to locate extrema along. Because the nature of Type 3 landmarks is so variable and dependent of such a wide variety of conditions Bookstein (1997) has recently revised his 1991 classification and termed this class of landmarks 'semilandmarks.' The category semi-landmarks includes the former Type 3 landmarks of Bookstein (1991) as well as the boundary coordinates used in outline morphometrics (e.g., Fourier analysis, eigenshape analysis, edgels).

These landmark classification systems are consistent with all types of coordinate-based observations that might be made on an object or form. Accordingly, previous distinctions between 'landmark-based' and 'non-landmark-based' (= outline-based) methods have now been abandoned. Perhaps even more importantly, this broadening of the types of data that can be treated by geometric morphometric methods allows geologists to capitalise on the widespread consensus among cognitive psychologists and pattern recognition researchers that the recognition of objects by their outlines is a fundamental part of the human visual system (see Koffka 1935; Attneave 1954; Marn 1976). Through geometric morphometrics geologists can quantify forms in a way that is detailed, analytically tractable, and natural.

The name 'eigenshape' is often taken to suggest that eigenanalysis represents the key the key computational step around which the entire method is constructed. This is only partially true. Within the eigenshape procedure eigenanalysis is used to achieve dimensionality reduction and to summarize the dominant, mutually independent modes of shape variation. However, this role is not unique to eigenshape analysis. Rather, it forms to basis for much numerical analysis and is widely employed by other morphometric methods (e.g., multivariate morphometrics, see Reyment et al. 1984; Reyment 1991; principal-partial-relative warp morphometrics, see Bookstein 1991) and composite data analytic approaches (e.g., PCA analyses of Fourier harmonic coefficients, see Rohlf 1986; Lestrel 1997). This again reflects the deep-seated similarity between eigenshape analysis and other coordinate-based morphometric procedures.

Lastly, the modeling capabilities of eigenshape analysis illustrated above are also not unique to this method. With sufficient understanding of the eigenanalysis procedure and algorithmic steps necessary for the determination of the shape function one can easily 'reverse-engineer' a modeling solution for any eigenanalysis-based decomposition that supports the representation of those results within a space that is fully analogous to the Euclidean space of the original measurements. By the same token, the results of an eigenshape analysis can also be portrayed using alternative graphical methods such as the thin-plate splines that are routinely employed in 'warp' analysis (see MacLeod in press a). The fact that, up to now, this has not been deemed either necessary or useful speaks more to the conceptual revolution that has occurred as a result of the morphometric synthesis than it does about computational difficulties.

In short, the forms of eigenshape analysis employed herein to specify, analyse, and model empirical shape space are not only closely related to other methods of coordinate-based morphometric analysis, they are completely synonymous with those other methods. Multivariate morphometric analysis of coordinate-point data, relative warp analysis of coordinate-point data, θ-ρ analyses of coordinate-point data, eigenanalysis of fourier harmonic coefficients, and eigenshape analysis are essentially variants of the same procedure, differing only in the methods traditionally employed to achieve inter-object registration, the graphical devices traditionally used to portray the results, and, of course, the jargon used to describe the procedure and its effect on the measurements. As a result, any or all of these methods could have been employed to yield comparable results. For this set of procedures the morphometric synthesis is essentially complete.

Implications of the ability to construct quantitative and semi-quantitative morphological models.—With such a powerful set of tools at the geologist's disposal for gaining control over patterns of shape variation, the next logical question is "to what geological end should these tools be put?" The simple answer is that, since geology concerns itself with shapes and shape comparisons on a routine basis, instruction in the theory and use of these tools should become part of every student's training and their application part of every editor and reviewer's expectations. The ability to quantify, summarize, ordinate, and model shape variation should free geologists (as well as biologists and other investigators of natural history phenomena) from their long-standing reliance on vaguely specified comparisons among forms and generalizations derived from qualitative 'tests'. In addition to the ordination and modeling capabilities reviewed herein, a large number of new statistical tests are also available to examine specific hypotheses in the light of various null models (see Rohlf 2000 and references therein for a review).

With respect to the main topic of this contribution, the ability to use morphometric methods to construct quantitative or semi-quantitative classification systems raises many interesting possibilities across the entire spectrum of geological sciences. Along with the refinement of currently established universal classification systems (examples 1 and 2 above) and the creation of local classification systems (example 3), this approach holds promise for helping to resolve many long-standing problems in the description of morphological data, the communication of morphological concepts, and the standardization of morphological nomenclature. In sciences (like geology) that depend on morphological analysis at virtually every level, this would be a represent a substantial improvement in the status quo.

While virtually any geological subdiscipline could supply many examples, nowhere are the problems inherent in maintaining and transmitting morphological concepts more apparent than in paleontology. All paleontological species and higher taxa are identified by their morphological structures. Because of this, an incredibly complex technical jargon has developed around each taxonomic group to represent this morphological data. The assimilation of these concepts from text-based descriptions represents a formidable hurdle in the training of new systematic paleontologists. This difficulty is exemplified by the fact that illustrations are ubiquitous in most paleontological publications because of the inadequacies of simple vocabulary to this task. Indeed, the notoriously low inter-worker reproducibility of paleontological data is, in part, the result of difficulties inherent in standardizing and communicating the complex morphological concepts required for consistent diagnoses (e.g., Zachariasse et al. 1978; Ginsburg 1997).

The descriptive and modeling capabilities of geometric morphometrics are capable of addressing these issues at a level of geometric sophistication commensurate with the complexities of the problem (see MacLeod in press b for examples). At the simplest level these techniques can be combined with end-member shapes to create illustrations of morphological characters that define the limits of character state variation in a complete and unambiguous manner. End-member shapes could range from constellations of points or outlines representing simple geometrical figures to highly complex combinations of points and outlines derived from the specimens themselves. Using the tools of contemporary morphometrics the complexity of the figures to be represented is, for the most part, no longer an issue. At the more sophisticated end of this spectrum the data projection tools of multivariate analysis and the complete range of classical and newly formulated shape statistics can be employed to probabilistically evaluate hypotheses of similarity or difference between groups of measurements taken from specimens or between specimen measurements and the variance ranges of *a priori*-defined shape classes.

Such an approach to morphological analyses—for all geological disciplines, not just paleontology—holds promise for substantially improving the quality of descriptive geological datasets, the consistency of class-based identifications, and, through these mechanisms, the accuracy of geological hypothesis tests. Moreover, because of the nature of the software currently being developed to implement these procedures, these advantages can be acquired by geologists without them having to make the commitments required of research-grade morphometricians; in the same way that classical statistical tests can be applied to geological data by geologists who are not also research-grate statisticians. All that is required is a willingness to understanding of the principles of contemporary morphometric analysis—which can be gained by reading any one of a growing number of textbooks and review articles on the subject—and access to appropriate hardware-software (see Acknowledgements for suggestions on the latter).

Summary

Qualitative and semi-qualitative shape classifications are encountered routinely in the geological sciences. These classifications are necessary because of the range of forms that must be recognized in order to pursue geological studies, their complex geometries, the lack of geometric-quantitative background of most geology students, and the proven utility of such classifications. However, qualitative and semi-qualitative geological shape classifications can be difficult to use for those unfamiliar with the geometric concepts these classifications are designed to illustrate. The tools of geometric morphometrics can be used to investigate the adequacy of qualitative and semi-qualitative geological shape classifications. Moreover, the modeling capabilities of many geometric morphometric methods (e.g., eigenshape analysis) can be combined with end-member exemplar shapes and used to improve existing geological shape classification systems.

Three example shape classification systems were investigated to illustrate the principles and procedures involved in morphometric shape classification analysis: the Powers (1953) sedimentological grain sphericity and roundness classification, the Wolfe

(1993) leaf-margin lobateness character, and the Small (1972) Val d'Hérens alpine valley profile data. These examples were selected on the basis of the diversity of their subject matter as well as the fact that they represent different types of geometric data. All three classifications were analyzed with the eigenshape morphometric method in either its standard or extended form (see MacLeod 1999) using comparable levels of digitization resolutions and representational fidelities.

In the Powers (1953) example, results of an eigenshape-based ordination of non-landmark-registered exemplar grains outlines within the three most important shape-similarity and shape dissimilarity eigenvectors (= eigenshapes) confirms the initial suspicion that these grains contain a significant proportion of shape variation that is extraneous to sphericity and roundness factors the scale was designed to illustrate. While the inclusion of such extraneous patterns of variation is all but unavoidable when real objects are selected as exemplars, the presence of such patterns represents a source of confusion and possible misspecification. Indeed, these eigenshape results suggest that the predominant patterns of shape variation recorded by the Powers (1953) exemplar outlines are not consistent with the traditional ordering of sphericity and roundness classes. However, by conducting a second eigenshape analysis of the four sphericity and roundness end-member shapes illustrated by Powers (1953) a more precise and heuristically accurate alternative sphericity-roundness classification matrix was constructed.

In the second example the Wolfe (1993) leaf-margin lobateness character was evaluated using an extended eigenshape sample scheme that registered the leaf shapes with landmarks at the leaf base and leaf tip. Aside from the difference in sample strategy this example differed from the grain-shape analysis in that the problem involved a lack (rather than an abundance) of intermediate forms that could be used to illustrate geometries near the shape-class boundaries. Results suggested that a ternary classification scheme was the most appropriate system to use in defining the differences between non-lobate, palmately lobate, and pinnately lobate leaf margins, that non-lobate leaf margins represents a very narrow area within the overall leaf margin shape space, and that intermediate leaf margin geometries between non-lobate and palmately lobate end-member shapes contain geometries that would be classified under completely different characters within the Wolfe (1993) system. This latter result brings up the question of the geometric independence of at least of the Wolfe (1993) characters as well as illustrating the power and utility of the morphometric approach to the creation of shape-classifications.

Finally, in the third example a series of glaciated Alpine valley profiles was evaluated using another extended eigenshape procedure that allows landmark-registered open curves to be quantitatively compared with one another. This example differed from the previous two not only in the type of figures analyzed and the disciplinary context of the analysis, but also in the generality of the investigation. Whereas the sedimentary-grain and leaf margin analysis were based on widely accepted universal icons, this valley study was a decidedly local in scope. Nevertheless, morphometric results once again revealed unexpected complexities, this time in the form of profile ordinations that did not conform to the V-shaped or U-shaped end-members of standard geomorphological classification.

In all three examples the use of geometric morphometric procedures was able to reveal the true nature of geometric patterns found among a series of forms that had not been fully appreciated. Moreover, the modeling tools available in geometric morphometric analysis were able to represent locations within abstract shape spaces in terms of the morphologies those locations might represent. This improves the ability with which such results can be communication to general audiences and facilitates the creation of more accurate qualitative and semi-quantitative classification system. Application of morphometric data analytic strategies through the geological sciences can result in a dramatic improvement in the quality of form-based geological hypothesis tests and interpretations, especially when coupled with rigorous statistical analyses.

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